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Reaction Field Induced Interatomic Forces Between Atoms in the Presence of a Strong Magnetic Field

B. Ritchie

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Lawrence Livermore National Laboratory

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Reaction field induced interatomic forces between atoms in the presence of a strong magnetic field

Burke Ritchie

University of California
Lawrence Livermore National Laboratory
Livermore, California 94550

It is shown that the reaction field induced in an atom by a strong magnetic field is of order $B\alpha^2$ in atomic units (for magnetic field strength B and fine structure constant α). The reaction field causes a dipole-dipole interatomic potential energy to exist between a pair of atoms of order $B^{\frac{3}{2}}\alpha^{\frac{7}{2}}$, such that B must be of order $\alpha^{-\frac{7}{3}}$ for the interatomic energy to be of order one atomic unit. B of this order corresponds to a field strength of 1.66×10^{12} G, which is within the regime of field strengths considered in recent studies of atoms and molecules in the presence of a strong magnetic field.

I. Introduction

It is well known that an ultrashort intense laser pulse passing through a plasma induces a strong reaction field in the wake of the laser [1]. The source of the reaction or "wake" field, as it is called, is the current induced in the plasma electrons in the direction of propagation of the laser and perpendicular to its direction of polarization by the Lorentz component of the force $\vec{F}_L = \frac{e}{c}\vec{\upsilon} \times \vec{B}$, where $\vec{\upsilon}$ is the collective velocity of the electrons modelled as a fluid and $\vec{B} = \vec{\nabla} \times \vec{A}$ is the magnetic field associated with the vector field $\vec{A}$ of the laser. In this paper we show that the current associated with the quantum mechanical motion of electrons bound in an atom, which vanishes in the case of the field-free atom but is non-vanishing in the presence of fields, is the source of a reaction magnetic field when an external magnetic field is applied to the atom. In turn the reaction field causes an interatomic potential energy to exist between a pair of atoms.

Atomic and molecular structure in the presence of a strong magnetic field [2-6] is a subject of high current interest both in the astrophysical [2-3,7] and theoretical atomic physics [2-6,8] communities. Yet to our knowledge the presence of reaction fields generated in the presence of a strong magnetic field appears not to have been hitherto investigated.

Normally such reaction fields are entirely negligible, with strength in atomic units of order $B\alpha^2$, where α is the fine structure constant and B is the field strength of the applied field. The leading contribution to the interatomic potential energy for a pair of atoms turns out to be of order $B^{\frac{3}{2}}\alpha^{\frac{7}{2}}$. Indeed the strength of the applied field in recent studies [2-3] of atoms and molecules in the presence of strong magnetic fields is of order $\alpha^{-\frac{7}{3}}$ and therefore sufficient to make the interatomic potential energy of order one atomic unit. Thus the structure of the individual atom is strongly modified by the applied magnetic field, as shown in [2-3], but in addition an induced or reaction magnetic field is generated, whose source depends on the quantum structure of the atom in the presence of the applied field, and the reaction field, although it is weak compared to the applied field, can nevertheless be the source of interatomic potential energies of comparable or higher magnitude than ordinary field-free electrostatic interatomic potential energies. The theoretical underpinnings of these assertions is given in Sections II and III below.

II. Theory

The nonrelativistic quantum mechanical current for a single-electron atom is,

$$\vec{j} = \frac{\hbar}{m} \mathrm{Im}\, \psi^{*} \vec{\nabla} \psi - \frac{e}{mc} \psi^{*} \psi \vec{A} \quad , \qquad (1)$$

where ψ is the Schroedinger wave function and $\vec{A}$ is the vector potential. The first term on the right side of Eq. (1) vanishes for the field-free atom due to the degeneracy of the magnetic sublevels of the atomic eigenstates. This degeneracy makes it possible to choose a real rather than a complex set of wave functions. The degeneracy is lifted in the presence of a magnetic field such that the first term is nonvanishing for states of nonzero m quantum numbers. For simplicity we present the analysis only for magnetic quantum number m = 0 states; however we have studied the case for |m| > 0 states and find that the leading contributions to the inter-atomic potential energy due to the reaction field are also of order $B^{\frac{3}{2}} \alpha^{\frac{7}{2}}$, as in the m = 0 case.

In the presence of an external magnetic field $\vec{B}$ the total vector potential can be written,

$$\vec{A} = \vec{A}_r + \frac{1}{2} \vec{B} \times \vec{r} \quad , \qquad (2)$$

where the first term on the right side of Eq. (2) is a reaction field calculated from Maxwell's equation,

$$\nabla^2 \vec{A}_r = -\frac{4\pi e}{c}\vec{j} \quad , \tag{3}$$

and the second term is the component of the total vector potential due to the external magnetic field. Substituting Eq. (1) (having dropped the first term on the right side, as appropriate for m = 0 states) into Eq. (3) and using Eq. (2), one can write Eq. (3) in the familiar Helmholtz form,

$$(\nabla^2 - \frac{\omega_p^2}{c^2})\vec{A}_r = \frac{2\pi e^2}{mc^2}\psi^* \vec{B}\times\vec{r}\,\psi \quad , \tag{4}$$

where ω_p is defined as the "plasma" frequency of the atomic cloud,

$$\omega_p = \sqrt{\frac{4\pi e^2}{m}\psi^* \psi} \quad , \tag{5}$$

in analogy to the well-known fundamental frequency for the collective motion of a plasma [9] in which $\psi^*\psi$ is replaced by n_e, the macroscopic electron density of the plasma.

Obviously from Eq. (4) the reaction field is absorbed as it spreads from a point of origin, in analogy to the absorption of an electromagnetic field by an "overdense" plasma when the frequency of the EM field is less than the frequency of the plasma [10]. In the present application the frequency of the EM field is obviously zero. We estimate the mean free path between absorptions as $(\omega_p/c)^{-1} = 4.48 \times 10^{-9}$ cm, where we have assumed a cylindrical probability density in Eq. (5) whose radius is the cyclotron radius for a field of 10^{12} G [2-3], namely $\hat{\rho} = \sqrt{\frac{\hbar c}{eB}} = 2.57 \times 10^{-10}\,\mathrm{cm}$. Since the mean free path is much larger than the cyclotron radius, we are justified in dropping the second term on the left side of Eq. (4), whereupon the solution of Eq. (4) is,

$$\vec{A}_r = -\frac{e^2}{2mc^2}\int d\vec{r}'\, \frac{\psi^* \vec{B} \times \vec{r}'\, \psi}{|\vec{r} - \vec{r}'|} \quad . \tag{5}$$

The consequences of this reaction field will be explored in the next section.

III. Interatomic potential between a pair of atoms due to the presence of the reaction field.

Let us calculate the interatomic potential energy between a pair of atoms due to the presence of the reaction field induced by the ultrastrong applied magnetic field. In Eq. (5) we assume that the wave function is the stationary eigenstate of the atom in the presence of the applied magnetic field, as calculated nonperturbatively in [2-3]. The change in the atomic Hamiltonian due to the presence of the reaction field is thus considered perturbatively with respect to the zeroth-order basis of [2-3].

The most obvious consequence of the reaction field given by Eq. (5) is the existence of dipole-dipole interatomic forces. This phenomenon is described as follows. With reference to Fig. 1 the field given by Eq. (5) is found at the position $\vec{r}$ of an electron bound in a neighboring atom. Then the inverse interelectronic distance $|\vec{r}-\vec{r}'|^{-1}$ in Eq. (5) is expanded in powers of the inverse distance R of the line joining the nuclei of a pair of atoms. For example for a pair of atoms whose internuclear vector $\vec{R}$ is parallel to the external field (Fig. 1), $|\vec{r}-\vec{r}'|^{-1} = |\vec{R}+\vec{r}''-\vec{r}'|^{-1}$ is expanded in terms of monopolar and dipolar contributions as follows,

$$|\vec{R}+\vec{r}''-\vec{r}'|^{-1} \cong \frac{1}{R} + \frac{\vec{R}\cdot\vec{r}'}{R^3} - \frac{\vec{R}\cdot\vec{r}''}{R^3} + \frac{\vec{r}'\cdot\vec{r}''}{R^3} - 3\,\frac{(\vec{R}\cdot\vec{r}')(\vec{R}\cdot\vec{r}'')}{R^5} \quad . \qquad (6)$$

The monopolar contribution to Eq. (5) vanishes by symmetry since the integrand has odd parity. Further for an applied magnetic field in the z direction and parallel to the internuclear vector, as in Fig. 1, all dipolar contributions of Eq. (6) vanish except the transverse components of the dipole-dipole contribution $\frac{\vec{r}'\cdot\vec{r}''}{R^3}$. We can then write the following equations for the two transverse components of the reaction field,

$$A_{rx} = \frac{e^2 B}{2mc^2} \frac{y''}{R^3} \int d\vec{r}' \psi^* y'^2 \psi \tag{7a}$$

$$A_{ry} = -\frac{e^2 B}{2mc^2} \frac{x''}{R^3} \int d\vec{r}' \psi^* x'^2 \psi \quad , \tag{7b}$$

where $R > \hat{\rho}'$ (where the range of the variables x' and y' in Eqs. (7) is taken to be that of the cyclotron radius $\hat{\rho}'$, as determined by the size of the atomic wave function in the presence of the applied magnetic field). Hence the transverse components of the total vector field [Eq. (2)] per atom are,

$$A_x = -\frac{1}{2} y B + A_{rx} \tag{8a}$$

$$A_y = \frac{1}{2} x B + A_{ry} \quad , \tag{8b}$$

where we have dropped the double primed label for the coordinates of the electron of the upper atom shown in Fig. 1. The contribution to the Hamiltonian is $(eA)^2/2mc^2$, and the leading contribution comes from the cross terms in Eqs. (8). Hence the first order energy shift of a pair of H atoms is,

$$\Delta E = -\frac{e^4 B^2}{4m^2c^4}\frac{S_x^2+S_y^2}{R^3} \tag{9a}$$

$$S_x = \int d\vec{r}\,\psi^* x^2 \psi \tag{9b}$$

$$S_y = \int d\vec{r}\,\psi^* y^2 \psi \quad . \tag{9c}$$

The interatomic energy given by Eq. (9a) depends in atomic units on $B^2\alpha^4$ times an atomic factor with dimensions of length. If we assume that this factor is of order of a cyclotron radius $\hat{\rho} = \sqrt{\frac{\hbar c}{eB}}$ (assuming the smallest possible value for R, namely the cyclotron radius), then the energy shift scales in atomic units as $B^{\frac{3}{2}}\alpha^{\frac{7}{2}}$, such that B must be of order $\alpha^{-\frac{7}{3}}$ for the interatomic energy to be of order one atomic unit. In physical units this corresponds to a field of $B_0\alpha^{-\frac{4}{3}} = 2.35 \times 10^9\,\alpha^{-\frac{4}{3}}$ G = 1.66×10^{12} G, where B_0 = $mec/\hbar a_0$ (for Bohr radius a_0) is the atomic unit of field strength.

From Eq. (9a) we note that the applied-field induced interatomic potential energy is attractive in nature and thus will reinforce the attractive R^{-6} dispersion energy between a pair of atoms. The latter is of course modified by the applied-field dependence of the atomic structures of the individual atoms.

IV. Summary and conclusions

In this paper we have shown that in the regime of a superstrong magnetic field ($B > 10^{12}$ G) [2-3], which exists on the surfaces of many neutron stars, a reaction field is generated whose strength is of the order of one atomic unit or greater. The main consequence of the reaction field is the existence of interatomic forces which for increasing applied field strength can become larger than ordinary field-free interatomic forces. In the ranges of magnetic field strengths $B > 10^{12}$ G and temperatures $T > 10^5$ K it is believed that bound atoms, molecules, and chains may make up important constituents of the neutron star atmosphere [11]; thus an applied-field induced interatomic potential energy would be expected to have an important influence on such collision-dependent atmospheric properties as gaseous viscosity and diffusion.

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Figure Captions

Figure 1. Coordinate system for a pair of hydrogen atoms in the presence of an applied magnetic field $\vec{B}$ directed along z. The nuclei are assumed to be fixed at the respective origins and are separated by the internuclear vector $\vec{R}$. The cylindrical radii $\hat{\rho}'$ and $\hat{\rho}''$ indicate the cyclotron radii of the two atoms transverse to the direction of the applied field.

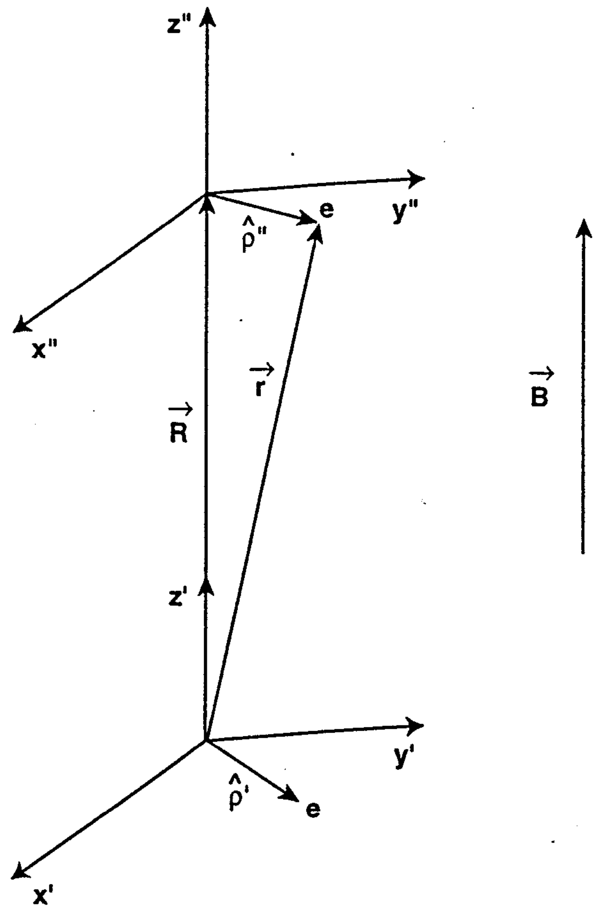

Figure 1

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